All Solutions to James Propp's Self-Referential Aptitude Test

Bob Smith
Sudley Place Software
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Introduction

The Self-Referential Aptitude Test[1] is a list of 20 multiple-choice questions created by Prof. James Propp[2], and is designed to test your ability to deal with self-reference. By that I mean a statement or question that refers to itself directly ("The answer to this question is") or indirectly through a chain of other statements/questions ("10. The answer to question 16 is", "16. The answer to question 10 is"), etc.

History

From Prof. Propp, "I created the S.R.A.T. for a party in 1993 or so...". It was also published in Math Horizons in Feb 2005[3]. He says that it took about 20-30 hours to create.

Notation

By Q4 #(A) ε {4,5,6,7,8} min (#(A)) is 4
By Q14 #(D) ε {6,7,8,9,10} min (#(D)) is 6
By Q3 #(E) ε {0,1,2,3,4} min (#(E)) Is 0

where #(X) means the count of the number of questions whose answer is (X), and @(X) is the current count of questions with answer (X). @(ABCDE) = 00000.

All Solutions

The following steps are an attempt to follow logic and logic only in order to ferret out all possible solutions to this amazing puzzle. No external information is used – for example, the answer to Q20 Is not assumed until the very end and even then all possible choices are considered.

Moreover, although the puzzle statement mentions the uniqueness of the answer, no assumption is made in that regard.

If you do feel the need to rely upon the uniqueness property, be careful how you interpret it. While the puzzle answer as a whole is unique, that doesn't mean that because any particular question admits multiple answers that the apparent duplicates can be discarded, or similarly, if one question has an answer which clearly is valid that is must be the correct answer. In other words, it's entirely possible for a question to have multiple valid answers, but that only one of them is correct.

1. **Q10 (A)** and **Q16 (D)** are the only compatible combination of answers to those two questions.
   @(ABCDE) = **1**00**1**0
2. Q13 allows us remove (A) from Q1, 3, 5, 7, 19.
   #(E) ε {1,2,3,4} min (#(E)) = 1
3. Q1 indicates there is at least one answer (B) in Q1-5, so Q11 (A) is out.
   min (#(B)) = 1
4. Q1 (B) can't be true as it would contradict Q1, so Q1 (B) is out which also eliminates Q2 (B).

5. Given that Q2 (B) is out, then Q7 (E) is out.
6. Given that the answer to Q10 is (A) and Q11 (A) is out, then Q2 (E) is out.
7. Given that Q2 (B) is out, then so is Q5 (B) which also eliminates Q1 (E).
8. Q6 and Q17 admit (B) and (D) as the only possible answers. Given that Q16 and Q17 cannot have the same answer, **Q17 (B)** and **Q6 (D)** must be correct.

   min (#(B)) = 2 @(ABCDE) = 1**1**0**2**0
9. Q2 and Q16 (D) eliminate Q15 (D), Q2 and Q17 (B) eliminate Q18 (B), and Q2 and Q6 (D) eliminate Q5 (D).
10. Eliminating Q15 (D) also eliminates Q12 (D).
11. If Q7 (B) is correct, that implies Q8 (E); if Q7 (C) is correct, that implies one of Q8 (A) or Q8 (E) is correct; and if Q7 (D) is correct, that implies one of Q8 (C) or Q8 (E) is correct. These limitations on Q8 from the possible choices of answers to Q7, eliminates Q8 (B) and Q8 (D).
12. min (#(A)) + min (#(E)) = 4+1=5, so Q8 (A) is out, which also means that Q7 (C) no longer implies Q8 (A).
13. If Q4 (E) is correct, than 8 + min (#(E)) = 9 which is greater than the choices in Q8, so Q4 (E) is out.
    #(A) ε {4,5,6,7}
14. If Q11 (E) is correct, than Q3 (B), Q4 (B), and Q9 (B) must all be correct. However, Q9 (B) implies Q11 (B) which is a contradiction., Thus Q11 (E) is out.
15. Given that Q11 (A) and Q17 (A) are out, so are Q13 (B) and (E). Also if Q13 (C) is correct, that's a contradiction, so Q13 (C) and (A) are out. If Q13 (A) is out, then so is Q9 (A). Given that Q9 (A) is out and Q10 (A) is correct, then Q2 (D) is out.
16. If Q9 (B) is correct, then Q11 (B) is too, and because we know from Q1 that at least one of Q3 (B) or Q4 (B) is correct that contradicts the answer Q11 (B), so Q9 (B) is out.
17. If Q11 (D) is correct, then all three (B) answers before Q11 (i.e., Q3, Q4, and Q7) are correct, which contradicts Q2, so Q11 (D) is out.

18. The only remaining answer to **Q13 is (D)**.
@(ABCDE) = 110**3**0

19. Given Q13 (D) is correct, then Q14 (D) is out (as per Q2) and **Q15 (A)** is correct, in which case Q14 (A) is out (as per Q2), in which case **Q12 (A)** is correct, and Q9 (C) is out.
#(D) ε {7,8,10} min (#(D)) = 7 @(ABCDE) = **3**1030

20. Given Q12 (A) is correct, so #(BCD) ε {2,4,6,8,10,12,14,16,18}. Moreover, min (#(BCD)) = 2+0+7=9, so #(BCD) ε {10,12,14,16,18}. As we already have three (A) answers,
#(BCD) ε {10,12,14,16}

21. The number of unanswered questions that admit (A) as an answer is 4 (Q2, Q4, Q18, and Q20). The number of answered questions with answer (A) is 3. If Q4 (D) is correct, then Q4 (A) is out which is a contradiction, so Q4 (D) is out.
#(A) ε {4,5,6}

22. Given that there are three questions already answered with (A) (Q10, Q12, Q15), then if Q4 (C) is correct, then the total number of questions with an answer of (A) is 6 and all of the unanswered questions that admit an answer of (A) (Q2, Q18, Q20) must be correct. If Q2 (A) is correct, then Q7 (D) is correct (as per Q2). If Q4 (C) is correct, then Q3 (B) is too (as per Q1). If Q18 (A) is correct, then #(B) = #(A) = 6. Also, there are two questions already answered with (B) (Q3, Q17) and the number of unanswered questions that admit an answer of (B) is (Q11, Q14, Q19). This makes the maximum number of questions with answer (B) to be 5 which is a contradiction, so Q4 (C) is out.
#(A) ε {4,5}

23. Assume that Q4 (A) is correct:
    a) Given that there are already three questions answered with (A), if Q4 (A) is correct, that makes four (A)s and so all of the remaining unanswered questions that admit an answer of (A) (Q2, Q18, Q20) are out.
    b) If Q4 (A) is correct, then so is Q3 (B) and Q1 (C) (as per Q1).
    c) If Q2 (A) is out, then Q2 (C) is correct as it's the only

remaining choice.
At this point there is a contradiction as Q1 (C) and Q2 (C) are both marked as correct, but (as per Q2), they can't both have the same answer, so Q4 (A) is out.
#(A) ε {5}

24. With Q4 (A) out, **Q4 (B)** is correct as it is the only remaining choice for that question.
@(ABCDE) = 3**2**030.

25. Given that Q4 (B) is correct, then Q3 (B) is out (as per Q2).
#(E) ε {2,3,4}

26. Given that Q4 (B) is correct (5 questions with answer (A)) and the answer to Q3 is 2, 3, or 4, then the number of vowels is 7, 8, or 9. However, the available answers to Q8 are 6 or 8, so the number of vowels must be 8, and **Q8 (E)** is correct.
@(ABCDE) = 3203**1**.

27. Given that Q8 (E) is correct (8 vowels) and Q4 (B) is correct (5 (A)s), then **Q3 (D)** (3 (E)s) is correct.
#(E) = 3 @(ABCDE) = 320**4**1.

28. Given that Q4 (B) is correct, the answer to **Q1 is (D)**.
@(ABCDE) = 320**5**1.

29. If Q14 (E) is correct (#(D) = 10), then along with the answer to Q8 (E) (8 vowels) answers (A), (D), and (E) account for 18 answers. Q4 (B) and Q17 (B) are correct and either Q7 (B) or Q11 (B) are correct, which accounts for 21 answers, so Q14 (E) is out.
#(D) ε {7,8}

30. Given that Q14 (E) is out, then Q9 (E) is too, leaving **Q9 (D)** as the only remaining choice for that question.
@(ABCDE) = 320**6**1.

31. Given that Q8 and Q9 have different answers, then Q2 (C) is out, leaving **Q2 (A)** as the only remaining choice for that question.
@(ABCDE) = **4**2061.

32. Given that Q2 (A) is correct, then so is **Q7 (D)**.
@(ABCDE) = 420**7**1.

33. As Q4 is the only question with answer (B) which precedes Q11, **Q11 (B)** is correct.
@(ABCDE) = 4**3**071.

34. Given that #(A) = 5 (as per Q4) and #(D) ε {7,8} (as per Q14), then Q18 (C) is out. Also as #(E) = 3 (from Q3), so Q18 (D) is out.

35. Assume that Q5 (C) is correct
*@(ABCDE) = 43**1**71*.
a) This implies that Q3 (C) is correct (from Q5)
*@(ABCDE) = 43**2**71*.
b) This implies that the number of vowels is 2 + 5 = 7 (as per Q3 (C) and Q4 (B)) which contradicts Q8 (E), so Q5 (C) is out.

35. The only remaining choice for **Q5 is (E)**.
@(ABCDE) = 4307**2**.

This leaves questions Q14, Q18, Q19, and Q20 as the only ones unanswered.

The count so far of the number of consonants is 3+0+7 = 10 plus one more for Q14 (although we don't know which consonant (B) or (C) it is) for a total of 11. From Q12 we know that the number of consonants is an even number and we know that the answers to Q18 are both vowels, so there is only one more answer which is a consonant and it must be to Q19 or Q20.

The constraints we have are as follows:

i. 1 of Q18, Q19, Q20 is answer (E) (as per #(E) = 3 with 2 already chosen)
ii. 1 of Q18, Q20 is answer (A) (as per #(A) = 5 with 4 already chosen)
iii. 1 of Q19, Q20 is answer (BCD) (as per Q12 (A) and the above reasoning on consonants)
iv. If Q18 (A), then #(A) = 5 = #(B)
v. If Q18 (E), then #(A) = 5 != #(B)

This yields several solutions, all of which ignore any external reasoning as to the answer to Q20:

- If the correct answer to Q20 is (BCD), then Q19 is (E) (as per constraint **iii.**), and Q18 (E) is out (as per Q2) leaving Q18 (A) as the only remaining choice. Q14 is (C) or (B) depending upon whether or not the consonant chosen for Q20 is (D). Given Q18 (A), then #(B) = 5 (2 more than we already have) and so the answers to Q14 and Q20 must both be (B). This fork represents a single consistent solution:
  Q14 (B), **Q18 (A)**, **Q19 (E)**, and **Q20 (B)**.
- If the correct answer to Q20 is (A), then the answer to Q19 is (BCD), and the answer to Q18 is (E) (as per constraint **ii.**). Given constraint **iv.**, Q14 and Q19 can't both be (B). This fork represents two consistent solutions:
  Q14 (B), **Q18 (E)**, **Q19 (C)**, **Q20 (A)**
  Q14 (C), **Q18 (E)**, **Q19 (D)**, **Q20 (A)**
- If the correct answer to Q20 is (E), then the answer to Q19 is (BCD), and the answer to Q18 is (A) (as per constraint **ii.**). Given Q18 (A), then #(B) = 5 (2 more than we already have) and so the answers to Q14 and Q19 must both be (B). This fork represents a single consistent solution:
  Q14 (B), **Q18 (A)**, **Q19 (B)**, and **Q20 (E)**.

In summary, there are four consistent solutions:

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                11111111112
  Q12345678901234567890
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1. DADBEDDEDABADBADBAEB
2. DADBEDDEDABADBADBECA
3. DADBEDDEDABADCADBEDA
4. DADBEDDEDABADBADBABE
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from which I choose #4 based upon the sentence it forms as well as the commonly understood answer to Q20. Prof. Propp says “the sentence DAD BEDDED A BAD BAD BABE was one of several hints to the identity of the movie "Fatal Attraction".”.

Author

If you find an error or a simplification, please contact me at bsmith@sudleyplace.com.

Online Version

This paper is an ongoing effort and can be out-of-date the next day. To find the most recent version, download http://www.sudleyplace.com/SRAT-Solutions.pdf.

References

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